

An introduction to separable equivalence

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Beginning note

k an algebraically closed field

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algebra finitely generated unital algebra over k

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$$F: \text{mod } A \longrightarrow \text{mod } B$$

$$G: \text{mod } B \longrightarrow \text{mod } A$$

$$\text{Id}_{\text{mod } A} \cong GF$$

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In particular $\text{Hom}_A(M, N) \cong \text{Hom}_B(FM, FN)$.

Example

Example (1)

If $A \cong B$ then A and B are Morita equivalent.

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Example (2)

Any algebra A is Morita equivalent to the $n \times n$ matrix ring $\mathbb{M}_n(A)$.

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Sketch proof.

Given the modules M and N , the functors

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provide an equivalence of the categories.

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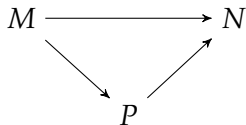
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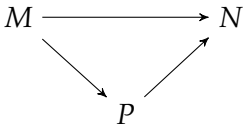
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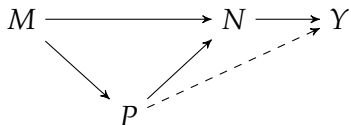
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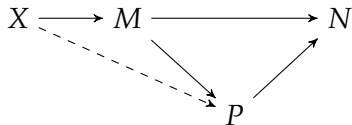
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Some groups

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Thus

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- ▶ kA_5 is Morita equivalent to $\mathbb{M}_n(kA_5)$ but not isomorphic
- ▶ kA_5 is stably equivalent to kA_4 but not Morita equivalent

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For k a field of characteristic 2, the group algebras kA_5 and $k[C_2 \times C_2]$ are separably equivalent.

Thus

- ▶ kA_5 is Morita equivalent to $\mathbb{M}_n(kA_5)$ but not isomorphic
- ▶ kA_5 is stably equivalent to kA_4 but not Morita equivalent
- ▶ kA_5 is separably equivalent to $k[C_2 \times C_2]$ but not stably equivalent

Complexity

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Proposition

Separable equivalence preserves complexity.

Question

If $\text{cx } A = \text{cx } B$ does this mean they are separably equivalent?

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Short answer

No.

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If $cx A = cx B$ does this mean they are separably equivalent?

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Slightly longer answer

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$$\text{cx } kG = \max \left\{ r \mid \overbrace{C_p \times \cdots \times C_p}^r \leq G \right\}$$

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No.

Slightly longer answer

For G a group

$$\text{cx } kG = \max \left\{ r \mid \overbrace{C_p \times \cdots \times C_p}^r \leq G \right\}$$

So $kC_p, kC_{p^2}, kC_{p^3}, \dots$ all have complexity 1.

New question

Is kC_{p^n} separably equivalent to kC_{p^m} for any $m \neq n$?

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Maybe.

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Slightly longer answer

$k[x]/(x^2)$ is not separably equivalent to $k[y]/(y^n)$ for $n \neq 2$.

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$$\text{Fun}(\underline{\text{mod}} \Lambda_n, \text{vec}) : \quad 1 \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} 2 \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} 3 \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} \cdots \begin{array}{c} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{array} n-1$$

with some relations



Ta.

